Lab 4: Circular Motion (M4)

Objectives

- Understand circular motion.
- Measure the radial force required to maintain an object in circular motion at constant speed.

Theory

An object is in **uniform circular motion** if it travels around a circular orbit (radius of the orbit = r) at a constant *speed*. However, the ***velocity*** (which is a vector) of the object does change its direction. Any changes in velocity mean that there is an acceleration called a **radial or centripetal acceleration**. To keep an object running a circular path, we need to apply an external force – called a **radial force**. The radial force value is equal to mass times the radial acceleration. In uniform circular motion, the radial acceleration is pointing towards the center of the circle, and it does change the velocity of the object, but it does not change its linear speed.

The angular velocity ω is defined as the rate of change of the angular displacement $\Delta\theta$.

The relation between linear (tangential) speed $v = \frac{\Delta s}{\Delta t}$, angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ and the period of revolutions T is given by the following equations.

$$|v| = r|\omega| = \frac{2\pi r}{T} \;, \tag{1}$$

$$\omega = \frac{v}{r} \quad \text{or} \quad |\omega| = \frac{2\pi}{\mathrm{T}} \tag{2}$$

In uniform circular motion, both speed v and angular velocity ω are constant.
In accordance with Newton's Second Law,

$$F = ma, \tag{3}$$

Because the radial acceleration is given by the following equation

$$a_r = \frac{v^2}{r} = \omega^2 r, \tag{4}$$

the radial force, which is a product of mass and radial acceleration, can be written as

$$F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r = (\omega^2 r)m \tag{5}$$

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The SI unit for angular velocity ω is radian per second (rad/s), although other units of angular velocity such as revolutions per minute (rev/min or *rpm*) are also popular. Keep in mind that:

$$One\ revolution = \Delta\theta = 360^\circ = 2\pi\ radians = 6.283\ radians$$

The conversion factor is equal to:

$$1\ rpm = \frac{2\pi\ radians}{60\ seconds} = 0.1047\ \frac{rad}{s}$$

The angular velocity is positive when the rotation is counterclockwise and negative when the rotation is clockwise.

Set-up:

The apparatus includes:

- "A"-shaped heavy base with a vertical rod.
- Rotating platform with two masses – one moving freely and one fixed to the platform. The rotating platform is located on the top of a DC motor.
- A force sensor hanging vertically with a thin string connecting the force probe and the “moving” mass on the rotating platform.
- An adjustable DC power supply for changing the angular velocity of the platform.

The whole set-up is shown below:

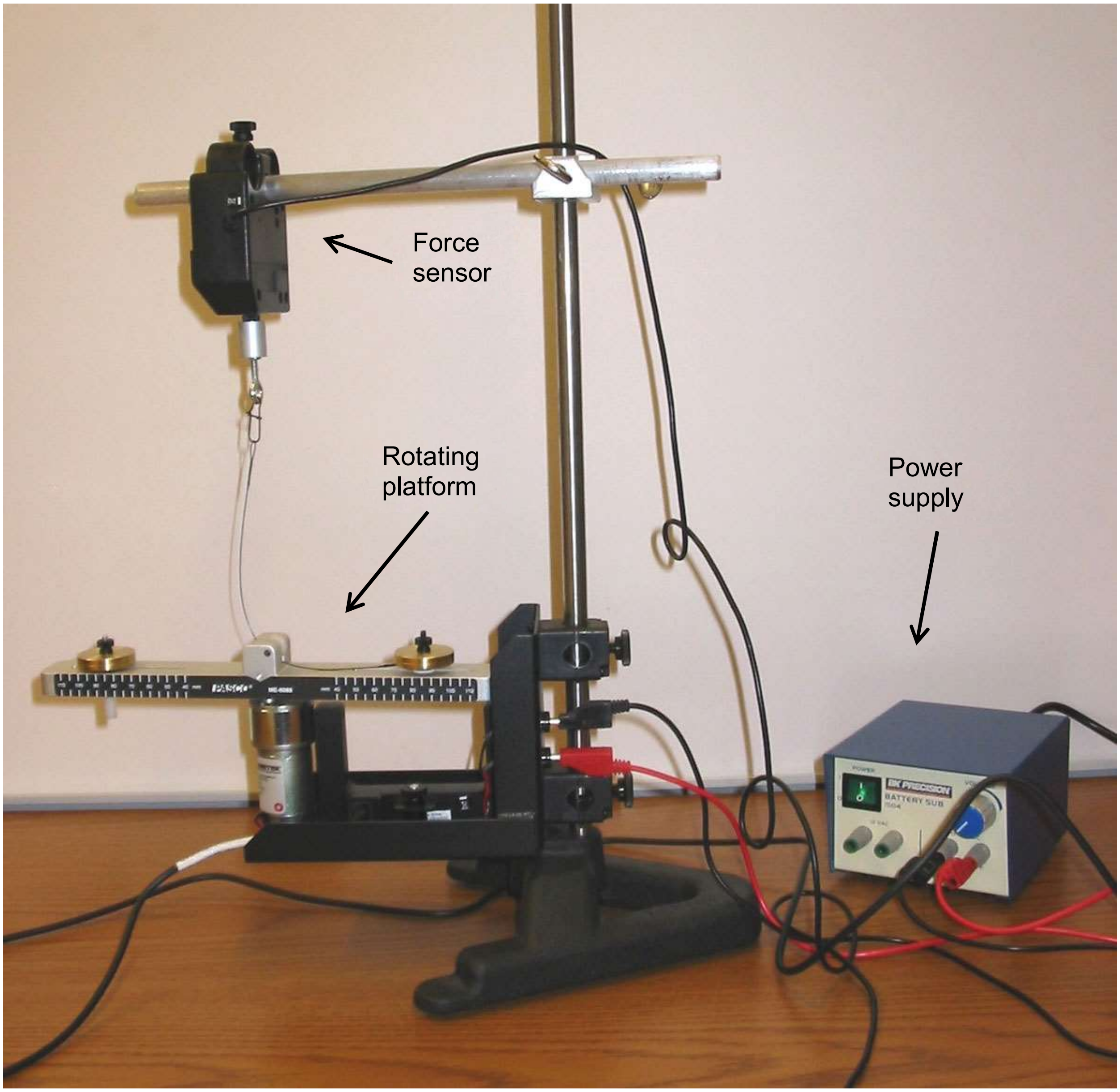


Figure 1.

The next picture shows the rotating platform with two masses (brass disks) and the photogate to measure the period of rotations.

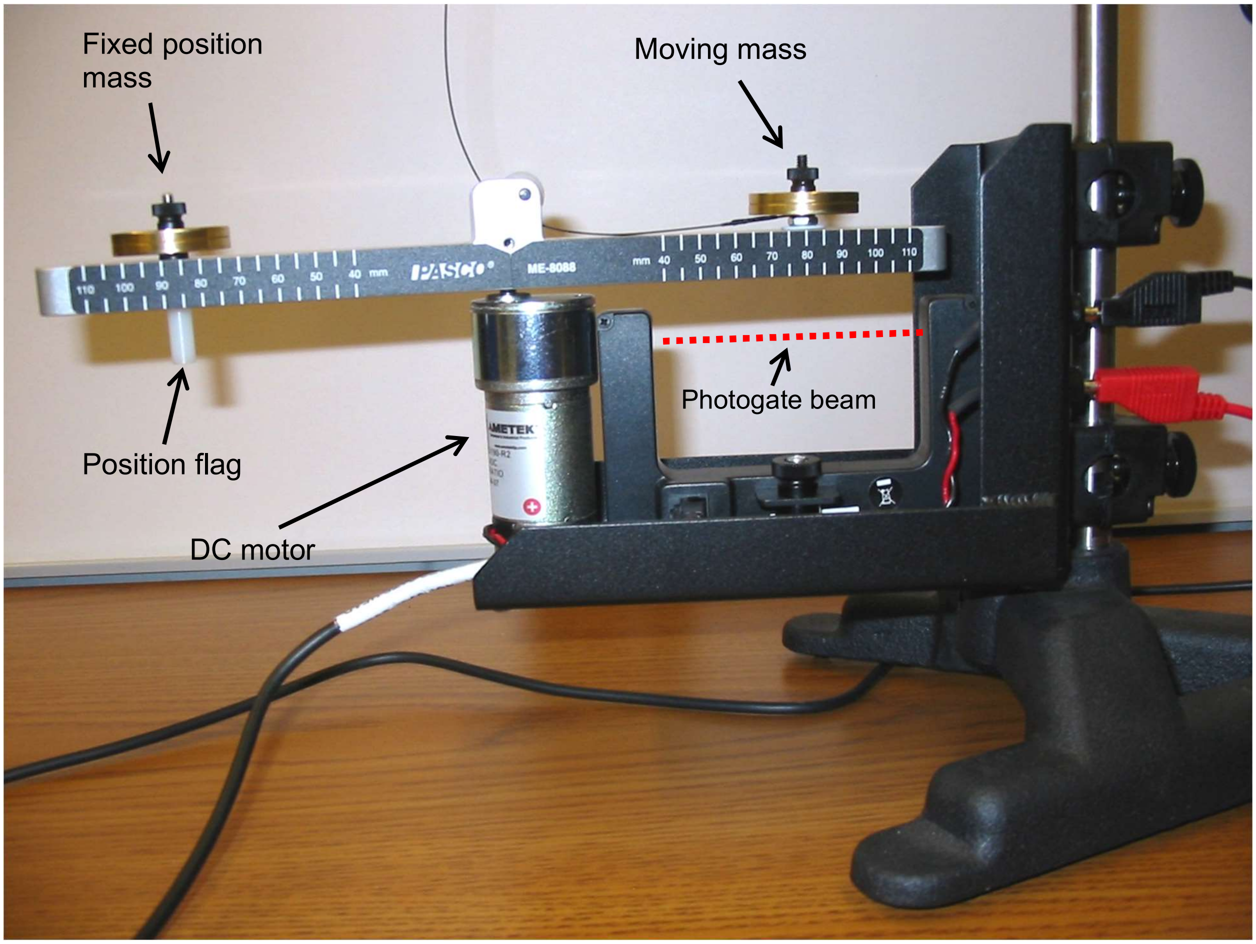


Figure 2.

The fixed position mass is needed as a counterweight for the moving mass to prevent excessive wobbling when platform is rotating. The values of these two masses should always be the same.

In each rotation, the position flag located below the fixed position mass crosses and briefly interrupts the photogate beam, sending a signal to the computer. Computer quickly measures the time interval between two consecutive photogate beam interruptions, which is simply equal to the period T of rotations. Next, computer calculates the angular velocity using equation (2): $|\omega| = \dfrac{2\pi}{T}$.

The radial force acting on the moving mass is measured directly using the force probe connected to the moving mass by a thin string (cable). The brass disks, which we use as the calibrated masses should always be above the string (cable). The whole setup allows you to continuously measure the radial force as a function of angular velocity.

Procedure:

Activity 1: Radial Force vs. Angular Velocity

In this activity, you will measure how the radial force s changing with angular velocity.

Login using your Purdue career account.

Download the Capstone files for experiment M4 using the same method as for previous labs and save them to desktop. Next, double-click on "M4 Activity 1" icon.

First, attach $m_0 = 20$ g masses (i.e., brass disks) on both the fixed position mass holder and the moving mass holder. Both holders should always have equal masses attached. The moving mass should be already connected to the force sensor with a thin string or a thin cable. Make sure that the string is always underneath the brass disks.

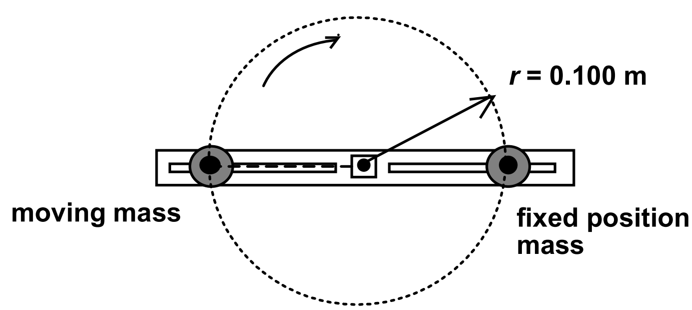
Each screw holding masses is $m_{screw} = 3.5$ g. Therefore, the rotating mass is going to be equal to the combined mass of the brass disk(s) and the screw. Do not include the fixed position mass in this calculation, because that one is not connected to the force sensor.

$$m = m_0 + m_{screw} = 20.0 \text{ g} + 3.5 \text{ g} = 23.5 \text{ g} = 0.0235 \text{ kg}$$

Check the position of both masses. The center of each mass, which is where the holding screw is located should be 100 mm away from the axis of rotation. When the rotating platform starts spinning, each mass will move in a circle. Since the distance between the mass and the axis of rotation is fixed at 100 mm, then the radius r of that circle is also fixed.

$$r = 100 \text{ mm} = 0.100 \text{ m}$$

Check that the moving mass can really move freely inside the groove. If the mass is not free to slide along the groove, then a significant frictional error could be present.



Make sure that cables are connected to red and black power supply terminals (i.e., DC terminals).

Next, **briefly push the "Tare" button** on the force sensor to zero it. You will have to push the "Tare" button before each measurement run – just before you start spinning the platform.

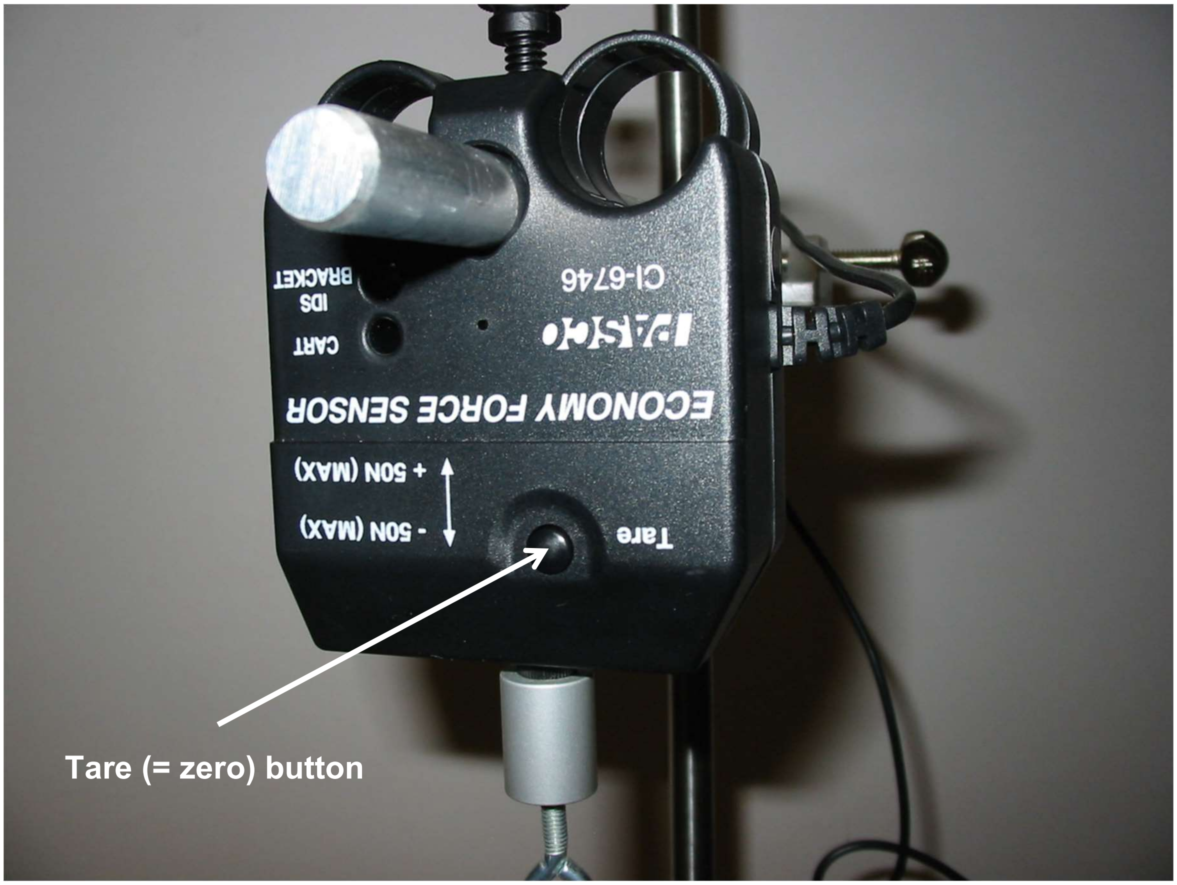


Figure 3.

Turn the power supply on. **Begin data recording** by clicking on the 

button. **Slowly**, but steadily increase the voltage from the power supply. After a short delay caused by static friction, the rotating platform will start spinning. Slowly increase the voltage until the angular velocity reach $\omega = 50$ rad/s. At that moment, **the data acquisition will automatically stop.** Turn the VOLTAGE knob counterclockwise back to zero to stop the rotating platform. The whole run from zero to $\omega = 50$ rad/s should take about 15-20 s. That should give you a reasonable number of data points. If the force number changes to red and there is no data recording, then it is an indication that you forgot to zero the force sensor before starting data acquisition.

Capstone was programmed to automatically fit a parabola to your data points continuously as you collect data. The parameters of that parabola should appear in a rectangular frame located inside the force versus angular velocity graph.

The theoretical formula given by equation (5) could be re-written as follows:

$$F_r = m\omega^2 r = (mr)\omega^2 \tag{6}$$

This is a quadratic equation. The computer calculates the following curve:

$$y = Ax^2 + B + C \qquad \text{or} \qquad F_r = A\omega^2 + B\omega + C \tag{7}$$

Therefore, the fit parameter A should be equal to the product of mass and radius.

$$A = mr \quad \text{and} \quad B = C = 0 \tag{8}$$

Calculate the mr value and compare it to the fit parameter A. Calculate the percent difference between these two values.

Repeat the same procedure for the following masses: m_0 = 30g and 40g. Find the parameter A of the fit for each value of mass. Remember to change both masses! Make sure to zero the force sensor by pushing the "Tare" button just before each run.

Print the graph with data and quadratic fit for m_0 = 40g and attached that to your lab report.

Activity 2: Radial Force vs. Mass

Quit *Capstone* application and go back to your Desktop. Next, double-click on "M4 Activity 2" icon.

Keep the 40g mass on the rotating platform (and another 40g as the fixed position mass). Zero the force sensor. Start data recording and adjust the voltage to keep the angular velocity at 30 rad/s. Record the value of force on the data sheets. If the force number changes to red and there is no data recording, then it is an indication that you forgot to zero the force sensor before starting data acquisition.

Increase the angular velocity to 40 rad/s and then to 50 rad/s. Record the radial force for each these velocities. The data acquisition should automatically stop at 52 rad/s (if not, then click on the "STOP" button). Repeat measurements for different values of mass: 30g, and 20g. Make sure to zero the force sensor before each run.

When finished with data recording, **prepare and print** a single graph (or three separate graphs) showing the value of radial force F (on vertical axis)$_r$ as a function of mass m (on horizontal axis) for each of the velocity values: ω = 30 rad/s, 40 rad/s and 50 rad/s. You should have three lines with three data points on each line. Note the slope change in these three graphs.

Describe how the radial force is changing with increasing mass. The theoretical formula for the radial force F_r vs. mass m is given by Eq. (5):

$$F_r = \left(\omega^2 r\right)m \quad \text{or} \quad y = ax + b$$

The radial force should be proportional to the mass of the rotation object. For each value of angular velocity ω find the straight line fit for F_r vs. mass m and find the proportionality coefficient (i.e., the slope) a.

For each value of angular velocity ω calculate the percent difference between the theoretical value of the slope $\omega^2 r$ and the observed value of the slope a.

Quit the program. Do not save any changes. **Complete the lab report and return it to the lab TA. Logout from your account.**

Make sure to complete the following tasks while in the lab room:

You must submit the answers to the prelaboratory questions online. (4 points)

1. Your completed Data Sheets. (4 points)
2. One printout from Activity 1 and one graph from Activity 2 (1+1 = 2 points)
3. Return the completed lab report to your lab TA.